High-Order/hp-Adaptive Multilevel Discontinuous Galerkin Methods With Applications in Fluid Dynamics

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Abstract

We present a discontinuous Galerkin (DG) multilevel method with adaptivity. The main advantage of this multilevel method is that the number of dimensions of the finite element space is independent on the presence of complicated or tiny features in the domain. In other words, even on a very complicated domain, an approximation of the solution can be computed with only a fistful of degrees of freedom. This is possible because two meshes are used: a fine mesh is used to describe the geometry of the domain with all its features, but the problem is actually solved on a coarse mesh that is, in general, too coarse to describe all the geometrical features of the domain. Unlikely other multilevel methods, this method does not perturb the problem, in the sense that the problem solved on the coarse mesh is always a discretization of the continuous problem, no matter how coarse the mesh is. The method itself is a hp-adaptive DG extension of composite finite elements (CFEs), introduced by S. Sauter a few years ago. Standard CFE methods are based on standard continuous Galerkin elements, which means that there are restrictions on the kind of boundary conditions that can be used. These limitations disappear by extending the method to DG elements.

We employed two types of error estimators: an explicit one and an implicit one based on a duality argument. In the latter case the element residuals of the computed numerical solution are multiplied by local weights involving the solution of a certain dual or adjoint problem. On the basis of the resulting a posteriori error bounds, we implemented an adaptive finite element algorithm to ensure reliable and efficient control of the error. The performance of the resulting hp-refinement algorithm is demonstrated through a series of numerical experiments.

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References

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