

# A FEniCS-based Finite Element Model for Thermo-Hydro-Mechanical Simulation in Porous Media

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## Abstract

The coupling between stress, flow and heat fields in porous media is an important subject with many geophysical applications. This work presents the design, implementation and verification of a fully coupled finite element model dealing with thermo-hydro-mechanical simulations. The mathematical model involves the stress equilibrium equation of the solid skeleton, the mass conservation of the fluid and the advection-diffusion equation for energy conservation. By making use of the automated solution techniques, we develop a readable and easily extendible code for this class of multi-physics problems. The code is developed with FEniCS [1], a recently developed platform for automated solution of partial differential equations through the finite element method. After implementation, we verify and validate the developed model by comparing the numerical results against commonly used benchmarks. Specifically, we use the Cryer problem [2] for hydro-mechanical coupling, the Horton-Rogers-Lapwood problem [3] for thermo-hydro coupling, and a benchmark from [4] for thermo-mechanical coupling. In all cases, our results are in good agreement with the benchmarks, and demonstrate the advantages of FEniCS for model development.

## References

1. M. ALNÆS AND J. BLECHTA AND J. HAKE AND A. JOHANSSON AND B. KEHLET AND A. LOGG AND C. RICHARDSON AND J. RING AND M.E. ROGNES AND G.N. WELLS. The FEniCS Project Version 1.5. *Archive of Numerical Software*, 3, 100, 2015, pp. 9-23.
2. C.W. CRYER. A Comparison of The Three-dimensional Consolidation Theories of Biot and Terzaghi. *The Quarterly Journal of Mechanics and Applied Mathematics*, 16.4, 1963, pp. 401-412.
3. D. NIELD AND A. BEJAN. *Mechanics of Fluid Flow Through a Porous Medium*. Springer New York, 2013.
4. A.Y. ROZHKO. Benchmark for Poroelastic and Thermoelastic Numerical Codes. *Physics of the Earth and Planetary Interiors* 171.1, 2008, pp. 170-176.