# Analysis of High-Speed Rotating Flow in 2D Polar ($R - \Theta$) Coordinate

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## Abstract

The generalized analytical model for the radial boundary layer in a high-speed rotating cylinder is formulated for studying the gas flow field due to insertion of mass, momentum and energy into the rotating cylinder in the polar (r - $\theta$) plane. The analytical solution includes the sixth order differential equation for the radial boundary layer at the cylindrical curved surface in terms of master potential ($\chi$), which is derived from the equations of motion in a polar (r - $\theta$) plane. The linearization approximation ((Pradhan & Kumaran, J. Fluid Mech., vol. 686, 2011, pp. 109-159); (Kumaran & Pradhan, J. Fluid Mech., vol. 753, 2014, pp. 307-359)) is used, where the equations of motion are truncated at linear order in the velocity and pressure disturbances to the base flow, which is a solid-body rotation. Additional assumptions in the analytical model include constant temperature in the base state (isothermal condition), and high Reynolds number, but there is no limitation on the stratification parameter. In this limit, the gas flow is restricted to a boundary layer of thickness ($Re^{(-1/3)}R$) at the wall of the cylinder. Here, the stratification parameter $A = \sqrt{m\Omega^2 R^2/(2k_B T)}$. This parameter $A$ is the ratio of the peripheral speed, $\Omega$R, to the most probable molecular speed, $\sqrt{2k_B T/m}$, the Reynolds number $Re = \rho_w \Omega R^2/\mu$, where $m$ is the molecular mass, $\Omega$ and $R$ are the rotational speed and radius of the cylinder, $k_B$ is the Boltzmann constant, $T$ is the gas temperature, $\rho_w$ is the gas density at wall, and $\mu$ is the gas viscosity. The major advantage of the present formulation is that it is not restricted to the asymptotic limit of high stratification parameter, and since we have used the conservative form of the compressible mass, momentum and energy conservation equations in deriving the generalized analytical model, obviously, the shock type solutions (Rankine-Hugoniot relations) are automatically satisfied. The analytical solutions are then compared with direct simulation Monte Carlo (DSMC) simulations and found good agreement (with a difference of less than 10%), provided the boundary conditions are accurately incorporated in the analytical solution. In a high speed rotating field we examine the mass flow rate through the stationary insert (intake tube). The simulations show that it varies significantly due to the equilibrium back pressure maintained at the rear end of the intake tube. The angular momentum loss of the rotating gas due to the stationary insert is studied for stratification parameter in the range $0.707 - 3.535$, and found significant loss which generates the secondary radial flow towards the axis, and it further excites the secondary axial flow. An important finding is that the stagnation pressure (no mass flow through the intake tube) is strongly affected by the wall gap, as well as with stratification parameter, indicating a strong coupling between the local temperature, density, pressure and velocity fields.

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