Non-Iterative Domain Decomposition Algorithms for Enhanced Mixed Finite Element Methods on Triangular Grids

Andrés Arrarás, Laura Portero
Department of Engineering Mathematics and Computer Science, Public University of Navarre, 31006 Pamplona (Spain)
andres.arraras@unavarra.es, laura.portero@unavarra.es

Abstract

In this work, we discuss numerical procedures for solving initial-boundary value problems in mixed form given by

$$\begin{aligned} p_t + \nabla \cdot \mathbf{u} &= f \qquad && \text{in } \Omega \times (0,T], \\ \mathbf{u} &= -K\nabla p \qquad && \text{in } \Omega \times (0,T], \end{aligned}$$

supplemented with suitable initial and boundary conditions, where $\Omega \subset \mathbb{R}^2$. In the framework of porous media flow, p represents the fluid pressure, $\mathbf{u}$ is the Darcy velocity, and K is a symmetric, positive definite tensor denoting the rock permeability divided by the fluid viscosity.

Following [2], we approximate the solution to the previous problem using the method of lines approach. The spatial discretization is based on an expanded mixed finite element method that reduces to a cell-centered pressure system by means of a suitable quadrature rule. This system is then integrated in time via a non-iterative domain decomposition splitting method with overlap. The resulting algorithm is convergent on smooth triangular meshes with six triangles per internal vertex (see [2]).

However, when applied to non-smooth triangulations, the accuracy of the method is observed to decrease. To avoid this behaviour, we introduce a technique, first proposed in [1] for elliptic problems, that enhances the original method by using Lagrange multipliers on certain element edges. The accuracy is then regained for so-called hierarchical meshes: i.e., those meshes obtained from an initial non-smooth coarse triangulation which is subsequently refined using a smooth refinement process inside each of the original coarse elements.

The fully discrete schemes for both the unenhanced and enhanced methods are derived in detail. Finally, a collection of numerical experiments illustrating the convergence behaviour of both algorithms is provided.

References

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