Verified Computations for Solutions to Nonlinear Heat Equations Based on Fractional Powers of a Positive Operator and the Evolution Operator

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Abstract

Let $J := (t_0, t_1]$ $(0 \leq t_0 < t_1 < \infty)$ and $\Omega = (0,1)^d \subset \mathbb{R}^d$ $(d = 1, 2, 3)$. In this talk we consider the following initial-boundary value problem of nonlinear heat equations:

$$\begin{cases} \partial_t u - \Delta u = u^p & \text{in } J \times \Omega, \\ u(t,x) = 0, & t \in J,\ x \in \partial\Omega, \\ u(t_0, x) = u_0(x), & x \in \Omega, \end{cases}$$

where $\partial_t u = \frac{\partial u}{\partial t}$, $\Delta = \frac{\partial^2}{\partial x_1^2} + ... + \frac{\partial^d}{\partial x_d^2}$, $1 < p < 1 + \frac{4}{d}$, $u_0 \in L^2(\Omega)$ is a given initial function. The aim of this talk is to present a numerical method for verifying existence and local uniqueness of a solution of this problem. Existence and local uniqueness of the solution in a Banach space $C\left(J; L^2(\Omega)\right)$ is proved in a neighborhood of an approximate solution $\omega(t,x)$, which is computed by numerical methods. A fixed-point formulation is derived by Tanabe-Sobolevskii's evolution operator. Using fractional powers of a positive operator, our main theorem gives a sufficient condition for enclosing the solution locally in time.

References

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