On the Convergence of Domain Decomposition Splitting Methods With Improved Initialization for Parabolic Problems

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Abstract

In this work, we deal with the numerical solution of parabolic problems of the form

$$u_t - \nabla \cdot (K\, \nabla u) + cu = f, \quad \text{in } \Omega \times (0, T], \tag{1}$$

together with certain initial and boundary conditions. Here $\Omega \subset \mathbb{R}^2$ is a bounded open domain with boundary $\partial\Omega$, $K(\mathbf{x}) \in \mathbb{R}^{2\times 2}$ is a symmetric positive definite tensor and $c(\mathbf{x})$ is a uniformly positive function on $\overline{\Omega}$. The entries of $K(\mathbf{x})$ and the functions $c(\mathbf{x})$ and $f(\mathbf{x}, t)$ are assumed to be sufficiently smooth.

Locally one-dimensional (LOD) methods, such as alternating direction schemes, constitute well-known alternatives to classical implicit time-stepping procedures when $K(\mathbf{x}) = k(\mathbf{x})I$, I being the 2×2 identity matrix. Nevertheless, such LOD methods often introduce a large splitting error that degrades the effectiveness of the resulting algorithms. In [1] and [3], the authors propose a modification of the right-hand side of certain LOD approaches which eliminates the contribution of this splitting error at a low extra computational cost.

The previous technique was extended in [2] to the case of considering domain decomposition time-splitting techniques. Problems that involve a full tensor K and/or require a non-Cartesian grid for their discretization can be handled in this case. The aim of this work is to present the convergence analysis of the methods proposed in [2], as well as to test them on a variety of general problems of type (1).

References

1. T. Arbogast; C.-S. Huang; S.-M. Yang. Improved Accuracy for Alternating-Direction Methods for Parabolic Equations Based on Regular and Mixed Fnite Elements. Math. Models Methods Appl. Sci. 17 (2007) 1279-1305.
2. A. Arrarás; L. Portero. Improved Accuracy for Time-Splitting Methods for the Solution of Parabolic Equations. Appl. Math. Comput. 267 (2015) 294-303.
3. J. Douglas Jr.; S. Kim. Improved Accuracy for Locally One-Dimensional Methods for Parabolic Equations. Math. Models Methods Appl. Sci. 11 (2001) 1563-1579.