Error Estimates With Explicit Constants for the Sinc Approximation Over Infinite Intervals

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Abstract

The Sinc approximation is a function approximation formula that attains exponential convergence for rapidly decreasing functions defined on the whole real axis. Even for other functions, the Sinc approximation is applicable when combined with a proper variable transformation. Stenger [1] proposed concrete variable transformations in some typical cases, e.g., polynomially decreasing functions defined on the whole real axis, polynomially decreasing functions defined on the right half real axis, polynomially decreasing functions defined on a finite interval. Furthermore, he gave theoretical error analysis in each case, and showed that the convergence rate of those approximations is $\mathrm{O}(\exp(-c\sqrt{N}))$. In the last decade, Mori–Sugihara [2] pointed out that the convergence rate can be improved by changing those variable transformations, and in fact, Tanaka et al. [3] showed that the convergence rate of those improved approximations is much higher: $\mathrm{O}(\exp(-c'N/\log N))$. Those error analyses are, however, incomplete for the purpose of computation with guaranteed accuracy, because some constants in the analyses are left unestimated. Recently, by revealing concrete forms of the constants, explicit error bound in the case of a finite interval has been given [4]. This study gives such explicit error bounds in the remaining cases.

References

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