

# Vector and Scalar Potential Formulation of a Mathematical Model for Induction Hardening With Non-linear Law for Magnetic Field

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## Abstract

We provide a derivation and an analysis of the mathematical model for induction hardening. We assume a non-linear relation between the magnetic field and the magnetic induction field. For the electromagnetic part, we use the vector and the scalar potential formulation.

The coupling between the electromagnetic and the thermal part is provided through the temperature-dependent electric conductivity and the joule heating term, the most crucial element, considering the mathematical analysis of the model. It functions as a source of heat in the thermal part and leads to the increase in temperature. Therefore, in order to be able to control it, we apply a truncation function.

Using the Rothe's method, we prove the existence of the global solution to the whole system. The nonlinearity in the electromagnetic part is overcome by utilizing the theory of monotone operators and the technique of Minty-Browder.

## References

1. D. HÖMBERG. A mathematical model for induction hardening including mechanical effects. *Nonlinear Analysis: Real World Applications* 5 (2004) 55-90.
2. M. BIEN. Global solutions of the non-linear problem describing Joule's heating in three space dimensions. *Math. Meth. Appl. Sci.* 28 (9) (2005) 1007-1030.
3. J. BARGLIK AND I. DOLEŽAL AND P. KARBAN AND B. ULRYCH. Modelling of continual induction hardening in quasi-coupled formulation. *Compel* 24 (1) (2005) 251-260.