Gene Mutations in Evolution Algebras by Means of Strong Isotopisms

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Abstract

Nonassociative algebras were introduced into genetics by Etherington [3] in order to endow Mendel's laws with a mathematical formulation. Specifically, he introduced genetic algebras like baric, train, gametic or zygotic algebras to deal with the sexual reproduction and the mechanism of inheritance of an organism by considering as an algebraic multiplication the fusion of a set of gametes into a zygote and the probability distribution of the gametic output of the latter. Holgate and Campos [2, 4] showed that certain families of genetic algebras are isotopic. The concept of isotopism was introduced by Albert [1] as a generalization of that of isomorphism. He indicated that two algebras $(A_1, \cdot)$ and $(A_2, \circ)$ are *isotopic* if there exist three regular linear transformations α, β and γ from A_1 to A_2 such that $\alpha(u) \circ \beta(v) = \gamma(u \cdot v)$, for all $u, v \in A_1$. Particularly, Holgate and Campos considered isotopisms of genetic algebras as a way to formulate mathematically the mutation of alleles in the inheritance process.

In order to deal with asexual reproduction processes, Tian and Vojtechovsky [5,6] introduced evolution algebras as a type of genetic algebra that makes possible to deal algebraically with the self-reproduction of alleles in non-Mendelian Genetics. Particularly, an n-dimensional algebra E over a field $\mathbb{K}$ is said to be an *evolution algebra* if it admits a basis $\{e_1, \ldots, e_n\}$ such that $e_i e_j = 0$ if $i \neq j$ and $e_i e_i = \sum_{j=1}^{n} a_{ij} e_j$, for some *structure constants* $a_{i1}, \ldots, a_{in} \in \mathbb{K}$. Each basis vector constitutes an allele of a given gene; the product $e_i e_j = 0$ for $i \neq j$ represents uniparental inheritance; and each structure constant a_{ij} constitutes the probability that the allele e_i becomes the allele e_j in the next generation.

In this work, we retake the original research of Holgate and Campos on isotopisms of genetic algebras. Specifically, we make use of distinct techniques in Computational Algebraic Geometry to deal with those aspects of the structure of evolution algebras that are preserved by strong isotopisms of the form (α, α, γ).

References

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