Multi-scale Kernels for Nyström Based Extension Schemes

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Abstract

Kernel based dimensionality reduction techniques, such as diffusion maps [1], have become a common tool for modeling high dimensional datasets. The main advantage of these methods is that the resulting low dimensional representation preserves important features of the data, such as the geometry of the original set. Typically, the low dimensional embedding representation is constructed from a given training set. The embedding coordinates, which capture the underlying process that drives the data, are then used for evaluating a continuous or a discrete function on the data. The next step is to extend the embedding coordinates for evaluating the function on the new data points. The geometric harmonics [2] is a kernel-based extension method for extending such low dimensional representations. It is closely related to the Nyström [3] extension scheme.

In Nyström based extensions, one constructs a kernel from the data and uses its spectral decomposition to form a basis. It is desirable that the constructed kernel contains most of the dataset's information in the few leading modes of its spectrum. In this work we propose a family of kernels, which are constructed as multi-scale combinations of Gaussian kernels, to be used within the Nyström and geometric harmonics framework. Such multi-scale kernels were suggested, for example, in the context of fluid-dynamics simulations [4]. In the context of image processing, these kernels are related to forward-backward diffusion processes [5], which preserve edge information while smoothing the data. We review their spectral properties and show that their first few modes capture more information compared to the standard Gaussian kernel and thus have an advantage for Nyström based extension schemes.

References

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