Verified High-precision Eigenvalue Bounds for Partial Differential Operators

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Abstract

We are considering the exact lower and upper eigenvalue bounds for eigenvalue problems of various differential operators. The motivation of such a research is to provide sharp and rigorous eigenvalue bounds which are needed in computer-assisted proof for the solution existence of nonlinear differential equations.

In the field of numerical computation, the finite element method(FEM) has been a successful tool to give nice approximation for eigenvalues. However, the classical usage of FEM remains in the level of approximate computing. That is, the exact lower and upper bounds for the eigenvalue are not available. In our recent research, a new algorithm is developed based on the finite element method, which can give strictly correct lower and upper bounds for eigenvalues [2].

Moreover, with combination with Lehmann-Goerisch's theorem, it is now possible to provide up sharp eigenvalue bound with relative error as small as 1E-10, even for domain with re-entrant corners, where there may exists singularities for the solutions [1,3].

The proposed method has been successfully applied in the eigenvalue problems of differential operators such like Laplacian, Biharmonic, Stokes, Steklov and so on. To have a glimpse of interesting computation results, please come to our session.

References

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