An Index-aware Parametric Model Order Reduction for Parametrized Quadratic DAEs

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Abstract

Modeling of sophisticated applications such as coupled problems arising from nanoelectronics can lead to parametrized quadratic differential algebraic equations (DAEs):

$$\mathbf{E}(\mu)\mathbf{x}' = \mathbf{A}(\mu)\mathbf{x} + \mathbf{x}^{\mathrm{T}}\mathbf{F}(\mu)\mathbf{x} + \mathbf{B}(\mu)\mathbf{u}, \quad \mathbf{y} = \mathbf{C}(\mu)\mathbf{x} + \mathbf{D}(\mu)\mathbf{u}, \tag{1}$$

where $\mathbf{x} = \mathbf{x}(\mu, t) \in \mathbb{R}^n$ is the state vector, the matrix $\mathbf{E}(\mu) \in \mathbb{R}^{n\times n}$ is singular for every parameter μ, $\mathbf{A}(\mu) \in \mathbb{R}^{n\times n}, \mathbf{B}(\mu) \in \mathbb{R}^{n\times m}, \mathbf{C}(\mu) \in \mathbb{R}^{\ell\times n}, \mathbf{D}(\mu) \in \mathbb{R}^{\ell\times m}$. The tensor $\mathbf{F}(\mu) = \left[\mathbf{F}_1^{\mathrm{T}}(\mu), \ldots, \mathbf{F}_n^{\mathrm{T}}(\mu)\right]^{\mathrm{T}}$ is a 3-D array of n matrices $\mathbf{F}_i(\mu) \in \mathbb{R}^{n\times n}$. Each element in $\mathbf{x}^{\mathrm{T}}\mathbf{F}(\mu)\mathbf{x} \in \mathbb{R}^n$ is a scalar $\mathbf{x}^{\mathrm{T}}\mathbf{F}_i(\mu)\mathbf{x} \in \mathbb{R}, i = 1, \ldots, n$. The vector $\mu \in \mathbb{R}^d$ encodes the parameter variations which may arise from material properties, system configurations, etc. In practice, these systems have very large dimension compared to the number of inputs and outputs. Despite the ever increasing computational power, simulation of these systems in acceptable time is very difficult, in particular if multi-query tasks are required. This calls for application of parametric model order reduction (PMOR). PMOR replaces (1) by a parametric reduced-order model (pROM): $\mathbf{E}_r(\mu)\mathbf{x}_r' = \mathbf{A}_r(\mu)\mathbf{x}_r + \mathbf{x}_r^{\mathrm{T}}\mathbf{F}_r(\mu)\mathbf{x}_r + \mathbf{B}_r(\mu)\mathbf{u}$, $\mathbf{y}_r = \mathbf{C}_r(\mu)\mathbf{x}_r + \mathbf{D}(\mu)\mathbf{u}$, with the matrices $\mathbf{E}_r(\mu) = \mathbf{V}^{\mathrm{T}}\mathbf{E}(\mu)\mathbf{V}$, $\mathbf{A}_r(\mu) = \mathbf{V}^{\mathrm{T}}\mathbf{A}(\mu)\mathbf{V}$, $\mathbf{B}_r(\mu) = \mathbf{V}^{\mathrm{T}}\mathbf{B}(\mu)$, and the tensor $\mathbf{F}_r(\mu) = \left[\mathbf{F}_{1_r}^{\mathrm{T}}(\mu), \ldots, \mathbf{F}_{n_r}^{\mathrm{T}}(\mu)\right]^{\mathrm{T}}$ is a 3-D array of r matrices $\mathbf{F}_{i_r}(\mu) \in \mathbb{R}^{r\times r}$, $r \ll n$. The projection matrix $\mathbf{V} \in \mathbb{R}^{n\times r}$ which is valid for all parameters μ in the desired range, and for arbitrary inputs $\mathbf{u}$, can be constructed using the projection based PMOR method in [2]. However, direct application of this approach to system (1), may produce parametric reduced-order models (pROMs) with index higher than the original model which might be inaccurate or are very difficult to solve. The same problem was already observed in linear DAEs, and could be eliminated by the index-aware MOR (IMOR) methods, proposed in [1]. We propose the extension of the IMOR method to parametrized quadratic DAEs which leads to accurate and easy to simulate pROMs. The robustness of the proposed approach is illustrated through PMOR on electro-thermal coupled models from industry.

References

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