Fast Intersection of Nonmatching Meshes Using Plűcker Coordinates

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Abstract

The extended finite element method (XFEM) represents one of the possible numerical methods for solving partial differential equations on complex geometries. In particular we are interested in the geometries coming from the geological applications involving 2D cracks and 1D wells in a 3D domain. The important challenge of the implementation of XFEM for the complex geometries is (a) an efficient identification of the intersecting pairs of elements, (b) an efficient computation of the actual intersection for the single element pair.

In our contribution we present a new algorithm solving the tasks (a) and (b) for the nonmatching meshes consisting of 1D, 2D, and 3D simplex elements. The algorithm is based on the methods used in the computer graphics, in particular for the task (b) we employ Plűcker coordinates to efficiently compute 1D-2D, 1D-3D, 2D-2D, and 2D-3D element intersections. For the 2D-3D case we carefully collect all the topological information coming from the performed Plűcker products to directly obtain the correct order and orientation of the edges of the intersection polygon.

For the task (a) we apply the broad first search on the connectivity graph of 2D and 1D submeshes to prolongate an initial 1D-3D and 2D-3D, respectively. intersection. Under assumption that the 2D and 1D submeshes are embedded in the 3D mesh (common case), we can make both the initialization as the search itself in the $O(N)$ time, where N is the number of the mesh elements. In this case, we also get directly the candidates for 1D-2D and 2D-2D intersection pairs. In other cases we construct a hierarchy of bounding boxes for the 3D elements leading to an $O(NlogN)$ algorithm.

References

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