Low-Rank Approximation of Stochastic Systems

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Abstract

In this contribution we focus on a development of an efficient solver for parametrized equations. The method is then used to solve a system of linear equations emerging from the discretisation of a stationary stochastic diffusion problem, e.g. the uncertainty propagation in groundwater flow environment. The stochastic part is taking place only in the uncertainty of a spatial distribution of the transport properties and is discretized with a Karhunen-Loeve Expansion (KLE) based on a generalized polynomial chaos (gPC). The spatial discretisation is treated with a standard finite elements and may itself constitute a large system of equations. Finally projecting the KLE to the same gPC basis leads to the tensor product scheme of a parametrized system of equations. Since the dimension of spatial and stochastic Galerkin subspace grows very fast, the standard solvers which are operating on a full tensor product of the solution, may become intractable due to the lack of computational resources. In order to keep the computation feasible even for a very large problems, one can solve the resulting system approximately on a smaller submanifold. This is basically achieved by separating the spatial and stochastic bases of the solution, exploiting its tensor product structure. The bases are then updated and compressed throughout the iteration process using the truncated singular value decomposition (SVD) to keep the number of bases as low as possible. The solutions of given examples are then compared with a high precission Monte-Carlo simulations.

References

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