Cuts for 3D Magnetic Scalar Potentials: Visualizing Unintuitive Surfaces Arising From Trivial Knots

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Abstract

A wealth of literature exists on computing and visualizing cuts for the magnetic scalar potential of a current carrying conductor via Finite Element Methods (FEM) and harmonic maps to the circle. By "cut" we refer to an orientable surface bounded by a given current carrying path (such that the flux through it may be computed) that restricts contour integrals on a divergence-zero vector field to those that do not link the current-carrying path, analogous to branch cuts of complex analysis. In our previous paper, we aimed to extend cuts for knotted geometries into undergraduate curricula via open-source software including GMSH and Python in order to allow students to compute and 3-D print surfaces [Stockrahm]. The exercises therein were intended to be a gateway to the intuitive study of near force-free magnetic fields and plasma physics. Here we extend these methods to broaden the ability of students to utilize free, readily available tools to communicate technically through visualization.

The present work is concerned with a study of a peculiar contour that illustrates topologically unintuitive aspects of cuts obtained from a trivial loop. Specifically, an unknotted curve that bounds only high genus surfaces in its convex hull is analyzed [Almgren]. The current work considers the geometric realization as a current-carrying wire in order to construct a magnetic scalar potential. We first realize cuts as level sets of harmonic maps associated with the trivial knot, noting they cannot be genus minimizing while having support lying in the convex hull of the (topologically trivial) conductor. Second, we produce a geometric configuration of currents that cannot possibly be a force-free magnetic field, observing there is no topological obstruction to having a near force-free magnetic field as a result of an ambient isotopy.

References

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