

# Verified Algorithm for the Modified Bessel Function of the Second Kind

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## Abstract

A numerical algorithm is presented for computing a rigorous value of the modified Bessel function of the second kind. The function  $K_\nu(x)$ , which is known as one of the solutions to the modified Bessel differential equation, can be described as a semi-finite integration. Particularly for  $0 < \nu < 1/2$ , the integral has a singularity at the edge of the interval of the integral. It has been shown that the double exponential formula is one of the most efficient methods for calculating an approximate value of such integrals [1]. However, a detailed explanation for obtaining the rigorous error bound for such integrals has not been clarified. In this talk, we present an efficient theorem about the error bound of the integrals based on the double exponential formula, and propose an algorithm so as to calculate verified results of the integrals. Numerical results are presented for showing the effectiveness of the proposed algorithm.

## References

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