Dimension-independent Boundary Operators for LAR of Cellular Models

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Abstract

The Linear Algebraic Representation (LAR) is a general and simple representation scheme for topology of geometric models, including simplicial, cuboidal and polytopal meshes, graphical and solid models, 2D and 3D images, mixed-dimensional BIM used in AEC applications. The domain of LAR scheme includes cellular models with non-convex and multiply connected cells. Its computer representation takes advantage of sparse characteristic matrices of cells, that are binary matrices storing the images of characteristic functions of cells as subsets of model vertices. The more interesting features of LAR [**?**] are simplicity, compactness, and great generality.

This talk will focus on the efficient construction of boundary and coboundary operators in the framework of the ongoing development of a novel python library, named `LarLib`, for dimension-independent computations with geometric and solid models and assemblies. The library is being currently used for the extraction of well-formed solid models from 3D biomedical images, and for the construction of virtual models of buildings and small/large built areas, for indoor mapping, IoT, and security applications. `LarLib` is being developed on top of `PyPLaSM`, a long-standing project for dimension-independent solid modeling [**?**].

A boundary operator is a linear map between linear spaces of *chains*, considered as subsets of cells. When its matrix is given, the $(d-1)$-boundary of every d-chain (any subset of d-cells) is computed by a single SpMV product of the operator, times the coordinate representation of the chain, i.e. its characteristic vector.

In this talk I will discuss several approaches to the computation of the sparse boundary matrix and its transpose coboundary matrix. In particular, I will distinguish between oriented and non-oriented operators, and between specialized implementations for simplicial, convex and general LAR complexes, including cellular decompositions with multiply-connected cells, i.e. with cells including any finite number of holes, that are especially useful for semantically-based computer modeling of buildings. The talk includes some joint work from a long-standing collaboration on LAR with Antonio DiCarlo and Vadim Shapiro.

References

1. A. Dicarlo and A. Paoluzzi and V. Shapiro. Linear algebraic representation for topological structures. *Comput. Aided Des. 46* (Jan. 2014), 269–274.
2. A. Paoluzzi and V. Pascucci and M. Vicentino. Geometric programming: A programming approach to geometric design. *ACM Transactions on Graphics 14*, 3 (1995), 266–306.