The SubIval Numerical Method for Computations of the Fractional Derivative in IVPs

Marcin Sowa
Silesian University of Technology
marcin.sowa@polsl.pl

Abstract

The concept of a fractional order derivative is a known topic in advanced mathematical analyses. It has been studied numerously (in various definitions) but apart from it being challenging in an analytical sense, the fractional order derivative has its usefulness in practice e.g. in control theory (where the fractional PID controller has been noticed to have improved stability properties), in electromagnetic field analyses (when modelling composite materials) and circuit theory (in modelling lossy coils and supercapacitors). The author is most interested in the latter, where coils and capacitors of fractional order provide the two simplest circuit elements applying the fractional derivative in their description. When solving linear problems (with typical source time functions) there are many possibilities on how to deal with the fractional order derivative. Some authors focus on obtaining the solutions analytically (this leads to formulae dependent on the Mittag-Leffler function), while others attempt to solve circuit problems with fractional order elements numerically. There have been many attempts at numerical methods that could generally allow to approximate the fractional order derivative. Generally in most cases these are constant-step methods. The paper presents the proposition of a numerical method, generally basing on a division into subintervals. For simpler future reference - the method is called SubIval for simpler future reference as it was first called the "subinterval-based method", when it appeared in [1]. The method allows variable time stepping and is also well described in sense of its implementation. It allows for accurate approximation of the fractional derivative as it bases on arbitrary polynomial interpolations in selected subintervals and exact formulae for monomial differentiation. The application of the method allows to obtain a convenient form similar to that yielded by backward differentiation formulae. The form can be used e.g. in implicit time stepping methodologies like, as shown in the paper, within a predictor-corrector scheme. When comparing it with results obtained via analytical-numerical methods (basing on the Mittag-Leffler function) it has been observed that the method displays very good accuracy. The method has been implemented in the C# language and put into an ActiveX DLL, which makes it available to use not only for C# programmers but also engineers that would like to use it in popular computation environments like Matlab or Mathematica.

References

1. M. Sowa. A subinterval-based method for circuits with fractional order elements. Bulletin of the Polish Academy of Sciences, Technical Sciences 62.3, 449-454. (2014).