On the Divergence Constraint in Mixed Finite Element Methods for Incompressible Flows

Volker John
Weierstrass Institute for Applied Analysis and Stochastics
john@wias-berlin.de

Abstract

The divergence constraint of the incompressible Navier–Stokes equations is revisited in the mixed finite element framework. While many stable and convergent mixed elements have been developed throughout the past four decades, most classical methods relax the divergence constraint and only enforce the condition discretely. As a result, these methods introduce a pressure-dependent consistency error which can potentially pollute the computed velocity. These methods are not robust in the sense that a contribution from the right-hand side, which influences only the pressure in the continuous equations, impacts both velocity and pressure in the discrete equations.

This talk reviews the theory and practical implications of relaxing the divergence constraint. Several approaches for improving the discrete mass balance or even for computing divergence-free solutions will be discussed: grad-div stabilization, higher order mixed methods derived on the basis of an exact de Rham complex, $H(\mathrm{div})$-conforming finite elements, and mixed methods with an appropriate reconstruction of the test functions. Numerical examples illustrate both the potential effects of using non-robust discretizations and the improvements obtained by utilizing pressure-robust discretizations.

This topic is joint work with Alexander Linke (WIAS Berlin), Christian Merdon (WIAS Berlin), Michael Neilan (Pittsburgh), and Leo G. Rebholz (Clemson).

References

1. Volker John and Alexander Linke and Christian Merdon and Michael Neilan and Leo G. Rebholz. On the divergence constraint in mixed finite element methods for incompressible flows. WIAS Preprint 2177, 2015.