An Operator Splitting Based Alternating Direction Implicit Method for Solving Multi-dimensional Helmholtz Equation

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Abstract

Numerical solution of multi-dimensional Helmholtz equation with high wave number remains a challenging computational task, mainly because of two reasons. First, due to the high wave number, it is required that the continuous Helmholtz equation should be discretized by a very fine grid, which leads to a huge linear algebraic system. It is even worse for the high wave number case, as a fixed number grid points per wavelength is required. The large size of the linear algebraic system makes it difficult to use direct method, in particular in higher-dimensional cases. Second, the discrete Helmholtz equation is a highly indefinite linear algebraic system which is extremely hard to be solved by classical iterative methods. For example, many existing iterative methods such as conjugate method converges slowly or not even converge. In this paper an operator splitting technique is used to factorize the 2D or 3D Helmholtz equation into a sequence of 1D Helmholtz equations, which is then directly solved by efficient tridiagonal solver. Numerical experiments are conducted and numerical results are compared with other popular methods, which demonstrated that the new method is accurate, efficient and efficient in solving the Helmholtz equation with high wave number.

References

1. E. Turkel and D. Gordon and S. Tsynkov. Compact 2D and 3D sixth order schemes for the Helmholtz equation with variable wave number. J. Comput. Phys. 232.1 (2013) 272-287..