Goal Oriented Time Adaptivity Using Local Error Estimates

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Abstract

When solving ODEs or PDEs, one is not always interested in the whole solution, but rather just a functional of the solution $\boldsymbol{y}(t)$. Here, we consider functionals of the form

$$J(\boldsymbol{y}) = \int_{t_0}^{t_e} j(\boldsymbol{y}(t))dt = \sum_{n=0}^{m} \int_{t_n}^{t_{n+1}} j(\boldsymbol{y}(t))dt,$$

with $j : \mathbb{R}^n \to \mathbb{R}$. Examples for this are determining the net-flow of mass in an engine or the total heat production of a system. The functional can also filter specific points or areas in a spatial discretization or give components different weights.

The current approach for controlling the error for the quantity of interest is the method of weighted residuals [2], which is used to estimate the global error in the quantity of interest. This method requires solving the given PDE forward in time and its adjoint problem backwards in time multiple times each, to reach a desired precision.

We instead propose a new approach using time-adaptive schemes [1] where the timesteps are determined using only the quantities that are relevant for $J(\boldsymbol{y})$. This way one may control the local errors in the time-discretization and thus the error $||J(\boldsymbol{y}) - J_h(\boldsymbol{y}_h)||$ of a numerical solution $\boldsymbol{y}_h$, using just one forward integration. Here $J_h(\boldsymbol{y}_h)$ is a discretization of $J(\boldsymbol{y}_h)$ using quadrature formulas.

$$J_h(\boldsymbol{y}_h) = \sum_{n=0}^{m} \Delta t_n \sum_{i=1}^{s} b_i \, j((\boldsymbol{y}_h)_n^{(i)}).$$

In the talk, an analysis of this method and numerical results showing strengths and weaknesses of the new method will be presented.

References

1. L. F. Shampine. Numerical solution of ordinary differential equations. Vol. 4. CRC Press, 1994.
2. R. Becker and R. Rannacher. An optimal control approach to a posteriori error estimation in finite element methods. Acta Numerica 2001 10 (2001) 1-102.