A New High Order Scheme for Navier-Stokes Equations in Irregular Domains

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Abstract

We are interested in high-order discretizations of the Navier-Stokes equations. The Navier-Stokes equations play a central role in modeling fluid flows. Here we focus on incompressible flows. It is well-known that this system may be represented in pure streamfunction formulation as follows.

$$\partial_t \Delta\psi + \nabla^{\perp}\psi \cdot \nabla\Delta\psi - \nu\Delta^2\psi = f(x, y, t),$$

where $\nabla^{\perp}\psi = (-\partial_y\psi, \partial_x\psi)$ is the velocity vector. These equations are supplemented with the no-slip boundary condition and with an initial condition.

We present a high-order finite difference scheme for Navier-Stokes equations in irregular domains. The scheme is an extension of a fourth-order scheme for Navier-Stokes equations in streamfunction formulation on a rectangular domain (see BenArtzi-Croisille-Fishelov [1]). The discretization offered here contains two types of interior points. The first is regular interior points, where all eight neighboring points of a grid point are inside the domain and not too close to the boundary. The second is interior points where at least one of the closest eight neighbors is outside the computational domain or too close to the boundary. In the second case we design discrete operators which approximate spatial derivatives of the streamfunction on irregular meshes, using discretizations of pure derivatives in the x, y and along the diagonals of the element.

We extend the fourth-order scheme (BenArtzi-Croisille-Fishelov [1]) to irregular domains. The strategy used here is to present the biharmonic operator $\partial_x^4 + 2\partial_x^2\partial_y^2 + \partial_y^4$ as a combination of pure fourth-order derivatives in the x, y and the diagonal directions $\eta = (x + y)/\sqrt{2}$, $\xi = (y - x)\sqrt{2}$. Indeed, Δ^2 can be expressed as

$$\Delta^2\psi = \psi_{xxxx} + 2\psi_{xxyy} + \psi_{yyyy} = \frac{2}{3}(\psi_{\eta\eta\eta\eta} + \psi_{\xi\xi\xi\xi} + \psi_{xxxx} + \psi_{yyyy}).$$

Then, the pure fourth-order derivatives may be approximated via a compact scheme using the values of the function and its directional derivatives. The convective term is teated in a similar manner, i.e., mixed third-order derivatives are written as combinations of pure third-order derivatives.

Numerical results show fourth-order convergence rate.

References

1. M. Ben-Artzi and J-P. Croisille and D. Fishelov. Navier-Stokes Equations in Planar Domains. Imperial College Press, 2013..