Fractional Fractal Continuum Fluid Flow Model for Heterogeneous Porous Media

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Abstract

In this work, a new fluid flow model formulation is presented. It describes the anomalous flow of a single-phase fluid within heterogeneous porous media of radial symmetry, and where classical Fickian-type models are not enough to explain neither the short-time nor the long-time behaviors[1]. The formulation encompasses fractal continuum and fractional concepts in order to capture both sub- and super-diffusive flow. Fractal continuum formulation assumes that porous medium does not completely fill the Euclidean space and as a consequence of that, line, surface and volume elements need to be properly mapped from fractal to Euclidean space [2]. This mapping process is done through a homogenization technique and allows using fractal elements in terms of Euclidean ones:

$$dV_D = c_3(D)dV, \tag{1}$$

$$dS_d = c_2(d)dS, \tag{2}$$

$$d\ell\alpha_i = c_1(\alpha)dx_i, \tag{3}$$

where dV_D, dS_d and $d\ell_\alpha$ are the fractal elements of volume, surface and line, respectively, and sub-index indicates their fractal dimensions. The Euclidean elements are dV, dS and dx_i. As a result, it is possible to represent super-/sub-diffusive flow without employing space-fractional derivative. The time-fractional derivative present in the model takes into account non-local effects on the fluid flow due to some relevant phenomena that occur at different time-scales. The effect of fractal model parameters is analyzed in order to characterize the anomalous diffusion and to determine which of them are feasible of retrieve from real data if they are available. Furthermore, the model was applied to cases of interest in reservoir engineering like constant terminal rate and constant terminal pressure. Results show the capability of the model presented here to be applied to pressure field tests or decline curve analysis.

References

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