K-measures and Discrete Laplacians

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Abstract

I introduce the boundary of (compactly supported) smooth k-vector fields as primary, and define the exterior derivative of differential k-forms via an integral duality. This is nicely consistent with the way in which boundary and coboundary are introduced in algebraic topology and discrete exterior calculus. The notion of boundary of k-vector fields extends naturally to general k-measures. Then, identifying each k-dimensional submanifold with boundary with the corresponding characteristic k-measure reconciles the different notions of boundary. In conclusion, both sharp and blurred boundaries are uniformly covered. In computational physics, this mathematical formulation is instrumental in constructing almost "perfect" discrete Laplacians [1], by renouncing—or better, weakening—the locality requirement. This paper builds on and integrates previous work done by the author in collaboration with F. Milicchio, A. Paoluzzi and V. Shapiro [2].

References

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2. A. DiCarlo and F. Milicchio and A. Paoluzzi and V. Shapiro. Discrete Physics Using Metrized Chains. Symposium on Solid and Physical Modeling. San Francisco, CA, October 5–8, 2009, ACM (2009), 135–145, ISBN/ISSN: 978-1-60558-711-0.