Adaptive Discontinuous Galerkin Method Based on the Dual Weighted Residual Estimation Technique

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Abstract

We deal with the numerical solution of time-independent nonlinear partial differential equations using the discontinuous Galerkin method. We employ the Dual Weighted Residual method (DWR), see e.g. [1], for a posteriori estimation of the computation errors. Unlike the usual error estimation methods, where the error is measured in a norm arising from the mathematical formulation, in DWR we choose the so-called target functional (representing some particular quantity of interest) at the beginning of the computation. Then, the error of the approximate solution is measured using this functional. Therefore, the whole adaptivity process can be driven by this quantity of interest which leads to a very efficient computation.

The DWR method comes from the classical residual a posteriori error estimation techniques (see e.g. [3]), but on top of that it weights the local contributions of the error indicators using the solution of the corresponding dual problem. Practically, this solution has to be approximated and its efficient computation has a great impact on the costs of the adaptive algorithm.

To minimize the computational effort we solve the dual problem with the same polynomial degree as the primal problem and then we reconstruct this function to a higher polynomial space using the approach described in [2]. Our goal is an efficient adaptive method for the solution of nonlinear PDEs, exploiting the information from the primal and dual problem not only for adaptive refinement of the computational mesh, but also to balance the algebraic errors. We confront our algorithm with other adaptive techniques and illustrate its performance on several numerical examples.

References

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